Solution to MHT CET – 2021

21^{st} September (Shift - 2)

Section I

PHYSICS

1. (C)

Pressure in side the first bubble $= P + \frac{4T}{r_1}$

Pressure inside the second bubble $= P + \frac{4T}{r_2}$

using the formula $PV = nR\theta$ θ = absolute temp.

we have : $\left(P + \frac{4T}{r_1}\right) \cdot \frac{4\pi}{3} r_1^3 = n_1 R' \theta$ (R' is molar gas constant)

$$\left(P + \frac{4T}{r_2}\right) \cdot \frac{4\pi}{3} r_2^3 = n_2 R' \theta$$

and $\left(P + \frac{4T}{R}\right) \cdot \frac{4\pi}{3} R^3 = (n_1 + n_2) R' \theta$

$$\therefore \left(P + \frac{4T}{R}\right) \cdot \frac{4\pi}{3} R^3 = \left(\frac{P + 4T}{r_1}\right) \cdot \frac{4\pi}{3} r_1^3 + \left(P + \frac{4T}{r_2}\right) \cdot \frac{4\pi r^3}{3}$$

$$\therefore \left(p + \frac{4T}{R}\right) R^3 = \left(P + \frac{4T}{r_1}\right) r_1^3 + \left(p + \frac{4T}{r_2}\right) r_2^3$$

on solving : $T = \frac{p(R^3 - r_1^3 - r_2^3)}{4(r_1^2 + r_2^2 - R^2)}$

2. (C)

Let the capacitance of capacitors are C initially.
So, afterward $C_1 = kC$ and $C_2 = C$
p.d. across capacitor C_2 is

$$V_2 = \frac{C_2 V}{C_1 + C_2} = \frac{kCV}{C + kC} = \frac{kV}{k+1}$$

3. (D)

4. (D)

Path difference for nth maximum is $n\lambda$.

5. (A)

$A = \pi r^2$, $\frac{A_2}{A_1} = \frac{1}{9}$

$\therefore \frac{r_2^2}{r_1^2} = \frac{1}{3}$

$h \propto \frac{1}{r}$ $\therefore \frac{h_2}{h_1} = \frac{r_1}{r_2} = 3$

$\therefore h_2 = 3h_1$

6. (A)

Rise in a capillary tube is given by

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$\therefore h \propto \frac{1}{g}$$

Smaller the value of g, greater will be h. Out of the given options, value g will be minimum in option (A).

7. (A)

Shunt resistance is given by

$$S = \frac{I_g G}{I - I_g}$$

In the first case : $3r = \frac{I_g G}{0.04 - I_g}$ ....(1)

In the second case : $r = \frac{I_g G}{0.08 - I_g}$ ...(2)

Dividing Eq(1) by Eq.(2) we get

$$3 = \frac{0.08 - I_g}{0.04 - I_g}$$

Solving, we get $I_g = 0.02$ A

8. (D)

Inductive reactance = R = $2\pi fL$

If L and f both are doubled, the inductive reactance will become 4R.

9. (C)

$g = \frac{GM}{r^2}$ where r is the distance from the centre of the earth

In the first case, $r_1 = R + R = 2R$

In the second case, $r_2 = R + \frac{R}{2} = \frac{3}{2}R$

$$\therefore \frac{g_2}{g_1} = \left(\frac{r_1}{r_2}\right)^2 = \frac{4 \times 4}{9} = \frac{16}{9}$$

$$g_2 = \frac{16}{9} g_1 = \frac{16}{9} \times \frac{9}{4} = \frac{4}{9} g$$

10. (A)

Velocity of sound is given by

$V = \sqrt{\frac{\gamma RT}{M}}$ $\therefore \frac{V_H}{V_{He}} = \sqrt{\frac{\gamma_H}{\gamma_{He}} \cdot \frac{M_{He}}{M_H}}$ (T is the same for both)

$$= \sqrt{\frac{7}{5} \times \frac{3}{5} \times \frac{4}{2}} = \frac{\sqrt{42}}{5}$$

11.(C)
Kinetic energy $k = \frac{1}{2}I\omega^2$

$\therefore \frac{K_2}{K_1} = \frac{I_2\omega_2^2}{I_1\omega_1^2} \quad \therefore \frac{1}{4} = \frac{I_2}{I_1}\cdot 4 \quad \therefore \frac{I_2}{I_1} = \frac{1}{16}$

$\frac{L_2}{L_1} = \frac{I_2\omega_2}{I_1\omega_1} = \frac{1}{16}\times 2 = \frac{1}{8}$

$\therefore L_2 = \frac{L_1}{8}$

12. (D)
The two equations can be written as

$$Y_1 = 2\sin 2\pi\left(\frac{4t}{0.2} - \frac{4x}{2}\right) = 2\sin 2\pi\left(\frac{t}{0.05} - \frac{x}{0.5}\right)$$

and $$Y_2 = 4\sin 2\pi\left(\frac{4t}{0.16} - \frac{4x}{1.6}\right) = 2\sin 2\pi\left(\frac{t}{0.04} - \frac{x}{0.4}\right)$$

Comparing with standard equation

$$Y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

We get, for the first wave,
$T = 0.05$ s and $\lambda = 0.5$ m
For the second wave,
$T = 0.04$ s and $\lambda = 0.4$ m
Hence their periods (hence frequencies) are not same.
Their wavelength are also not same.

For first wave velocity $= \frac{\lambda}{T} = \frac{0.5}{0.05} = 10$ m/s

For second wave velocity $= \frac{0.4}{0.04} = 10$ m/s

Hence velocity in same,

13. (C)
It is a balanced Wheatstone bridge. Hence no current flows through the galvanometer and it can be removed from the circuit. 5Ω and 1 Ω resistances are in series. Their equivalent resistance is $5 + 1 = 6\ \Omega$. 12.5 Ω and 2.5 Ω resistances are also in series. Their equivalent resitance is 15 Ω. 6Ω and 15 Ω resistances are in parallel. If I_1 and I_2 are the currents in them then

$6I_1 = 15\ I_2 \qquad \therefore\ I_2 = 0.4I_1$

$I_1 + I_2 = 2.1$ A $\qquad \therefore\ I_1 = \frac{2.1}{1.4} = 1.5$ A

14. (C)

Kinetic energy $k = \frac{p^2}{2m}$ where p is the momentum

$\therefore\ p^2 = 2mk$ or $p = \sqrt{2mk}$

$\therefore\ p \propto \sqrt{m}$

Hence bodies having greater mass will have greater momentum. Hence p and Q will have greater momentum compared to R.

15. (C)

Pressure $P = \frac{1}{3}\rho c^2$ $\therefore$ $c = \sqrt{\frac{3p}{\rho}}$

$\therefore$ $\frac{C_1}{C_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{16}{1}} = 4$

16. (C)

Fringe width $X = \frac{\lambda D_1}{d_1} = \frac{\lambda D_2}{d_2}$

$\therefore$ $\frac{d_2}{d_1} = \frac{D_2}{D_1}$ $\therefore$ $2 = \frac{D_2}{D_1}$

$\therefore$ $D_2 = 2D_1 = 2 \times 75 = 150$ cm

17. (C)

The two NAND gates whose two inputs are joined together behave like NOT gates.
The truth table can be written as

A	B	y_1	y_2	y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

We see that the output y is '1' if A or B or both are '1'. Hence it behaves as OR gate..

18. (A)

Change in momentum in each collision is [mv − (−mv)] = 2mv
Hence change in momentum in 5 collisions is 5 × 2 mV = 10 mV

19. (A)

In isochoric process, volume remains constant and hence no work is done. Hence statement (A) is incorrect.

20. (D)

Energy required to raise to satellite of m to a height h is equal to change in its potential energy.

$$\therefore \quad W = -\frac{GMm}{R+h} + \frac{GMm}{R} = \frac{GMmh}{(R+h)R} \qquad ...(1)$$

The energy of a satellite moving in a circular orbit is given by

$$E = \frac{GMm}{2(R+h)} \qquad ...(2)$$

$$\therefore \quad \frac{W}{E} = \frac{2h}{R}$$

21. (D)

$q = 4 \times 10^{-7}$

$q = Ne$

$N = \frac{q}{e} = \frac{4 \times 10^{-7}}{1.6 \times 10^{-19}} = 2.5 \times 10^{12}$

22. (B)
Centripetal acceleration $a=\frac{v^2}{r}$

For hydrogen atom, $V\propto\frac{1}{n}$ and $r\propto n^2$

$\therefore\ a\propto\frac{1}{n^4}\quad\therefore\ \frac{a_2}{a_5}=\left(\frac{5}{3}\right)^4=\frac{625}{81}$

23. (C)
The diode is forward biased. The potential difference between P and Q is $(3-(-5))=8$ V.

$I=\frac{V}{R}=\frac{8}{2\times10^3}=4\times10^{-3}\,A$

24. (C)

25. (B)
The kinetic energy per mole $E=\frac{3}{2}RT$

$\therefore\ R=\frac{2E}{3T}$

26. (C)
Maximum potential energy is attained at the highest point which gets converted into kinetic energy at the lowest point.

$h=2-0.75=1.25\,m$

$\frac{1}{2}mv^2=mgh$

$\therefore\ v^2=2gh=2\times10\times1.25=25$

$\therefore\ v=5\,m/s$

27. (A)
According to the Lenz's law, the induced current is such that it opposes the change in flux which causes it. Hence the induced current produces a field which opposes the field due to the magnet. For this it should flow in anticlockwise direction.

28. (D)
The wave number of the last line of the Balmer series is given by

$\frac{1}{\lambda}=\frac{R}{4}=\frac{10^7}{4}=25\times10^5\,m^{-1}$

29. (C)

$\mu_g=1.5=\frac{3}{2}\qquad\mu_w=1.33=\frac{4}{3}$

$\therefore\ {}_w\mu_g=\frac{\mu_g}{\mu_w}=\frac{3}{2}\times\frac{3}{4}=\frac{9}{8}$

$\sin C=\frac{1}{{}_w\mu_g}=\frac{8}{9}$

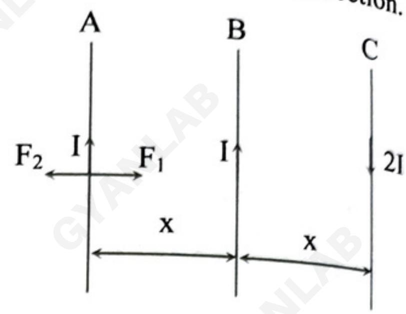
$\therefore\ C=\sin^{-1}\left(\frac{8}{9}\right)$

30. (D)

Currents in A and B are in the same direction. Hence force F_1 exerted by B on A will be attractive (towards B). Current in A and C are in opposite directions. Hence force F_2, exerted by C on A will be repulsive (away from C). Thus F_1 and F_2 are opposite in direction.

$$F_1 = \frac{\mu_0}{2\pi}\cdot\frac{I_1 I_2}{x}\cdot L = \frac{\mu_0}{2\pi}\cdot\frac{I^2}{x}\cdot L$$

$$F_2 = \frac{\mu_0}{2\pi}\cdot\frac{2I^2}{2x}\cdot L = \frac{\mu_0}{2\pi}\cdot\frac{I^2}{x}\cdot L$$



$\therefore$ F_1 and F_2 have some magnitude.

$\therefore$ $F_1 = -F_2$

31. (D)

In the given circuit, one end of each inductor is joined together. Similarly, the other end of each inductor is joined together. Hence the three inductors are connected in parallel. Their equivalent inductance is given by

$$\frac{1}{L} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$

$\therefore$ $L = 2$ H

32. (A)

The body acquired velocity V when it falls through a height h, starting from rest.

$\therefore$ $V^2 = 2gh$

$\therefore$ $h = \dfrac{V^2}{2g}$

If it falls further and attains velocity 3V and if the total height through which it falls is h', then

$(3V^2) = 2gh'$

$\therefore$ $9V^2 = 2gh'$

$\therefore$ $h' = \dfrac{9V^2}{2g} = 9h$

$\therefore$ $h' - h = 9h - h = 8h$

33. (C)

For a lens $\dfrac{1}{f} = (\mu' - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

where $\mu' = \dfrac{\mu_g}{\mu_\ell} = 1$

$\therefore$ $\dfrac{1}{f} = 0$ or $f = \infty$

34. (D)

$(K.E.)_1 = h\nu_1 - h\nu_0$ ...(1)

$(K.E.)_2 = h\nu_2 - h\nu_0$ ...(2)

Dividing Eq.(1) by Eq.(2) :

$$\frac{(K.E.)_1}{(K.E.)_2}=\frac{1}{k}=\frac{v_1-v_0}{v_2-v_0}$$

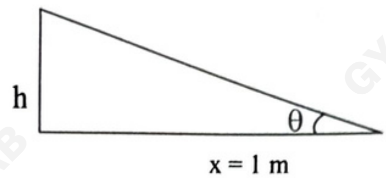
Solving for v_0 we get :

$$v_0=\frac{kv_1-v_2}{k-1}$$

35. (D)

$$\tan\theta=\frac{1}{20}=\frac{h}{1}$$

$$\therefore\ h=\frac{1}{20}\text{m}=0.05\text{ m}=5\text{ cm}$$



36. (B)

The moment of inertia of the upper ring about its diameter is given by

$$I_1=\frac{MR^2}{2}$$

The moment of inertia of the two lower rings about a tangent in their plane is given by

$$I_2=I_3=\frac{3}{2}MR^2$$

Total moment of inertia I = $I_1+I_2+I_3$

$$=\frac{MR^2}{2}+\frac{3}{2}MR^2+\frac{3}{2}MR^2=\frac{7}{2}MR^2$$

37. (B)

$$i=50\sin 100\pi t$$

$$\therefore\ 25=50\sin 100\pi t$$

$$\therefore\ \frac{25}{50}=\sin 100\pi t\quad\text{or}\quad \frac{1}{2}=\sin 100\pi t$$

$$\therefore\ 100\pi t=\frac{\pi}{6}\qquad\therefore\ t=\frac{1}{100}\text{s}$$

38. (B)

De–Broglie wavelength is given by

$$\lambda=\frac{h}{\sqrt{2mqv}}$$

If λ_1 and λ_2 are de–Broglie wavelengths of proton and alpha particle then

$$\frac{\lambda_1}{\lambda_2}=\sqrt{\frac{m_2q_2}{m_1q_1}}=\sqrt{4\times 2}=\sqrt{8}=2\sqrt{2}$$

39. (B)

Let R be the thermal resistance of each rod when connected in series, the total resistance will be

$$R_s=2R$$

Whey they are connected in parallel, the effective resistance will be

$$R_p=\frac{R}{2}$$

$\therefore \quad \frac{R_p}{R_s} = \frac{1}{4}$

In parallel combination, the resistance becomes one–fourth and hence time will also become one–fourth. $\therefore$ Time taken $\frac{8}{4} = 2\,s$

40. (C)

The periodic time of a pendulum is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$\therefore \quad T \propto \sqrt{\ell}$

Hence to increase the periodic time, length has to be increased. The periodic time is independent of mass and amplitude.

41. (B)

$$2 \times \left[\frac{1}{2\ell_1}\sqrt{\frac{T}{m}}\right] = \frac{\nu}{4\ell_2}$$

$$\frac{1}{0.5}\sqrt{\frac{50}{m}} = \frac{320}{4 \times 0.8}$$

$\therefore \quad m = 0.02$ kg/m

$\therefore$ Total mass of the string

$= 0.02 \times 0.5$ kg $= 10$ gm

42. (D)

The dark band between the 4th and 5th bright band in the 5th dark band.

For 5th dark band, the path difference is

$$\left(5 - \frac{1}{2}\right)\lambda = 4.5\lambda = 4.5 \times 6000 = 2.7 \times 10^{-4}\,\text{cm}$$

43. (A)

The standard equation of a wave can be written as

$$y = A\sin(kx + \omega t)$$

Speed of wave $V = \frac{\omega}{k} = \frac{60}{3} = 20$ m/s

Also, $V = \sqrt{\frac{T}{m}}$

$\therefore \quad m = \frac{T}{V^2} = \frac{0.4}{400} = 10^{-3}\ \text{kg m}^{-1}$

44. (D)

The charge stored by the charged condenser is

$$Q = C_1V_1$$

When the uncharged condenser is connected in parallel, the effective capacitance becomes $(C_1 + C_2)$

Hence the potential $V_2 = \frac{Q}{C_1 + C_2} = \frac{C_1 V_1}{C_1 + C_2}$

45. (C)
In a step up transformer, the current in the secondary is less than the current in the primary.

46. (C)
The gyromagnetic ratio of the electron

$$\frac{M}{L} = \frac{e}{2m} \quad \text{or} \quad M = \frac{eL}{2m}$$

47. (A)

$$V = V_{max} = \omega A = \frac{2\pi}{T} \cdot A$$

If A is tripled and T is doubled, V_{max} will become $\frac{3}{2}$ times or 1.5 times.

48. (B)
Volume of 8 smaller drop = Volume of the bigger drop

$$\therefore \; 8 \times \frac{4}{3}\pi r^3 = \frac{4\pi}{3} R^3$$

$$\therefore \; 2r = R \text{ or } r = \frac{R}{2}$$

Excess pressure $\Delta P_s = \frac{T}{2r}, \; \Delta P_B = \frac{T}{2R}$

$$\therefore \; \frac{\Delta P_B}{\Delta P_S} = \frac{r}{R} = \frac{1}{2}$$

49. (D)

50. (D)
Magnetic field inside the solenoid is given by

$$B = \frac{\mu_0 NI}{L}$$

$$\therefore \; \frac{\phi}{A} = \frac{\mu_0 NI}{L}$$

$$\therefore \text{ Magnetic moment, } NIA = \frac{\phi L}{\mu_0}$$

$$= \frac{1.5 \times 10^{-6} \times 0.6}{4 \times 3.14 \times 10^{-7}}$$

$$= 0.75 \text{ Am}^2$$

CHEMISTRY

51. (A)

At constant volume, $\Delta V = 0$

$\therefore \quad W = -P\,\Delta V \quad = 0$

According to first law of thermodynamics,

$\Delta U = Q + W$

$\therefore \quad \Delta U = Q \quad = x\ J$

52. (A)

Consider, oxidation state of I = x

$I_3^- \quad \Rightarrow 3x = -1$

$\therefore \quad x = -\frac{1}{3}$

53. (C)

For BCC unit cell, n = 2

$\therefore$ Total number of atoms in 1.8×10^{20} unit cells $= 2 \times 1.8 \times 10^{20}$

$= 3.6 \times 10^{20}$ atoms

54. (A)

$$\underset{\text{Propan-1-ol}}{CH_3-CH_2-CH_2-OH} \xrightarrow[623K]{Al_2O_3} \underset{\text{Propene (A)}}{CH_3-CH=CH_2} \xrightarrow[\text{(ii) } H_2O,\ \Delta]{\text{(i) conc. } H_2SO_4} \underset{\text{Propan-2-ol (B)}}{H_3C-\underset{\substack{|\\OH}}{CH}-CH_3}$$

55. (B)

1 mole of CH_4 = 16 g of CH_4 = 22400 cm^3 at STP

56. (D)

$[OH^-] = 1 \times 10^{-12}\ mol\ dm^{-3}$, $[H^+] = ?$

Product of hydrogen ion concentration and hydroxide ion concentration is equal to 1.0×10^{-14}.

$[H^+] \times [OH^-] = 1 \times 10^{-14}$

$\therefore \quad [H^+] = \frac{1\times10^{-14}}{1\times10^{-12}} = 1 \times 10^{-2}$

$= 0.01\ mol\ dm^{-3}$

57. (A)

58. (A)

Phenol form ester by reaction with carboxylic acid.

$$C_6H_5OH + CH_3COOH \xrightleftharpoons{H+} C_6H_5O-CO-CH_3 + H_2O$$

59. (B)

60. (B)

$$\underset{\text{Methyl bromide}}{CH_3-Br} + 2Na + \underset{\text{n-propyl bromide}}{Br-CH_2-CH_2-CH_3} \xrightarrow{\text{dry ether}} \underset{\text{Butane}}{CH_3-CH_2-CH_2-CH_3} + 2NaBr$$

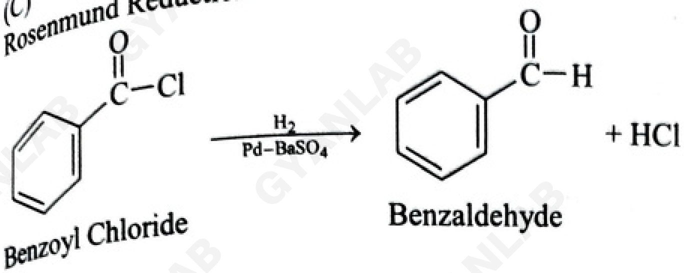
Butane is cross coupling product while ethane and hexane are self-coupling products.

Hence, propane is not formed in this mixture.

61. (A)

62. (B)
Glucose isomerase is useful in conversion of glucose to sweet-tasting fructose.

63. (C)
Rosenmund Reduction



64. (A)
$P_1^0 = 32$ mm Hg, $n_1 = 20$ mol, $n_2 = 2$ mol

$$\therefore x_1 = \frac{n_1}{n_1 + n_2} = \frac{20}{20+2} = 0.909$$

$$P_1 = P_1^0 x_1 = 32 \text{ mm Hg} \times 0.909$$

$$= 29.1 \text{ mm Hg}$$

65. (B)
$(CH_3CH_2)_2NH$ is secondary amine which is strong base and has lowest pK_b value.

66. (C)

67. (D)
$\wedge = 230\ \Omega^{-1}\ cm^2\ mol^{-1},\ k = 0.0115\ \Omega^{-1}\ cm^{-1}$

$$\wedge = \frac{1000k}{c}$$

$$\therefore c = \frac{1000k}{\wedge} = \frac{1000\,cm^3\,L^{-1} \times 0.0115\,\Omega^{-1}cm^{-1}}{230\ \Omega^{-1}\ cm^2\ mol^{-1}}$$

$$= 0.05 \text{ mol } L^{-1}$$

68. (D)

$$[PtBr_2(NH_3)_4]Br_2 \longrightarrow [PtBr_2(NH_3)_4]^{2+} + 2Br^-$$

69. (C)
Chemical properties of interstitial compounds are similar to the parent metal.

70. (A)
Sodium acetate is a salt of weak acid, acetic acid and strong base, sodium hydroxide. Therefore, it is basic solution with pH greater than 7.

71. (A)
68% of the total volume in bcc unit lattice is occupied by atoms and 32% is empty space or unoccupied volume.

72. (B)

$CH_3-CH_2-O-CH_2-CH_3$ (Ethoxy ethane) and $CH_3-O-CH_2-CH_2-CH_3$ (Methoxy propane)

are metamers of each other.

73. (C)

$$N_{2\,(g)} + 3H_{2\,(g)} \longrightarrow 2NH_{3\,(g)}$$

$$\Delta n_{(g)} = n_2 - n_1 = 2 - 4 = -2$$

$$\Delta H - \Delta U = \Delta n_{(g)} RT = -2RT$$

74. (C)

$M = 100\ g\ mol^{-1}$, $a = 400\ pm = 4 \times 10^{-8}\ cm$

For BCC structure, n = 2 atoms

$\rho = ?$

$$\rho = \frac{Mn}{a^3 N_A}$$

$$= \frac{100\,g\,mol^{-1} \times 2\,atoms}{(4\times10^{-8})^3\,cm^3 \times 6.022\times10^{23}\,atoms\,mol^{-1}}$$

$$= \frac{200}{64\times10^{-24}\times6.022\times10^{23}}$$

$$= 5.18\ g\ cm^{-3}$$

75. (A)

Solubilities of NaBr, NaCl and KCl changes slightly with temperature.

76. (D)

$[Co(NH_3)_6]^{3+} \Rightarrow Z = 27,\ X = 3,\ Y = 12$

EAN of $Co^{3+} \Rightarrow Z - X + Y = 27 - 3 + 12 = 36$

77. (A) **78.** (C)

79. (D)

$pH = 9.95$, $c = 0.05\ M$

$$pOH = 14 - pH = 14 - 9.95 = 4.05$$

$$pOH = -\log_{10}[OH^-]$$

$$\log_{10}[OH^-] = -4.05$$

$$= -4 - 0.05 - 1 + 1$$

$$= -5 + 0.95 = \bar{5}.95$$

$$[OH^-] = \text{antilog } \bar{5}.95$$

$$= 8.913 \times 10^{-5}\ M$$

80. (A)

The reaction of aryl halide with alkyl halide and sodium metal in dry ether to give substituted aromatic compounds is known as Wurtz-Fittig reaction.

81. (C) 82. (C)

83. (C)

t = 10 hour, $[A]_0 = 1.0$ M, $[A]_t = 0.25$ M

For first order reaction,

(i) $k = \frac{2.303}{t}\log_{10}\frac{[A]_0}{[A]_t} = \frac{2.303}{10\,h}\log_{10}\frac{1.0}{0.25}$

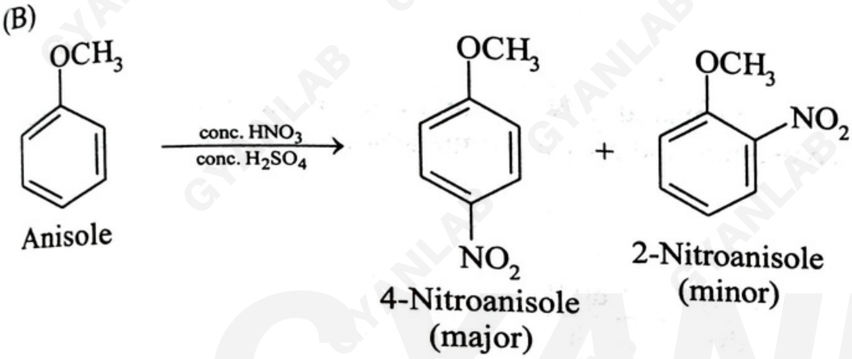
$= \frac{2.303}{10\,h}\log_{10}4 = \frac{2.303\times 0.6020}{10\,h}$

$= 0.139\ h^{-1}$

(ii) $t_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.139\ h^{-1}}$

$= 4.98$ hour ≈ 5 hours

84. (B)



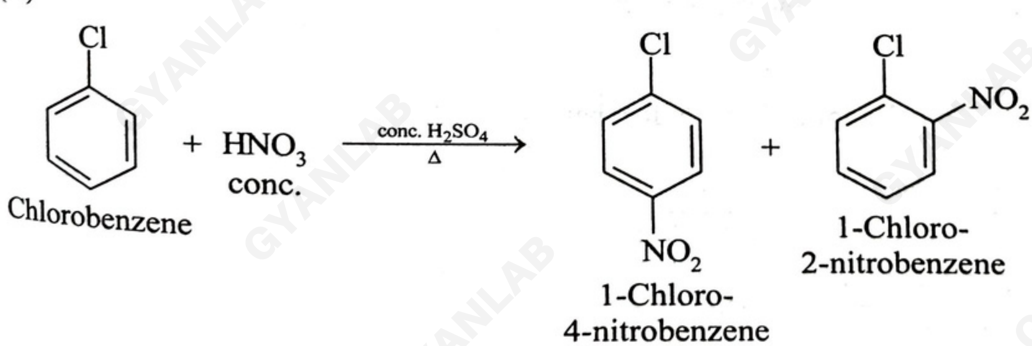
85. (D)

Ozone depletion is a major environmental problem because it increases the amount of UV radiation that reaches earth's surface.

86. (C)

Tetrafluoroethylene is polymerized by using free radical initiators such as hydrogen peroxide or ammonium persulphate at high pressure.

87. (A)



88. (B)

For glucose solution, m = 0.1 m, $T_b = 100.16$ °C

$\therefore \Delta T_b = T_b - T_b^\circ$

$= 100.16 - 100 = 0.16$ °C

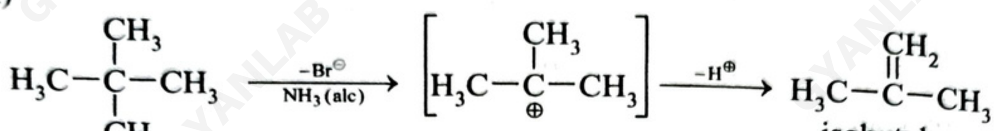
$$K_b = \frac{\Delta T_b}{m} = \frac{0.16\ °C}{0.1\,m} = 1.6\ °C/m$$

For sucrose solution, $m = 0.5\ m$, $K_b = 1.6\ °C/m$

$$\Delta T_b = K_b \times m$$
$$= 1.6\ °C/m \times 0.5\ m = 0.80\ °C$$

$$\Delta T_b = T_b - T_b^\circ$$
$$T_b = \Delta T_b + T_b^\circ$$
$$= 0.80 + 100 = 100.80\ °C$$

89. (A)



tert-Butyl bromide

isobutylene (2-Methylpropene)

90. (B)

Cl_2 is easily liquefiable gas as compared to N_2, O_2 and H_2. Hence, it adsorbed to greater extent at similar conditions of temperature and pressure if the adsorbent remains same.

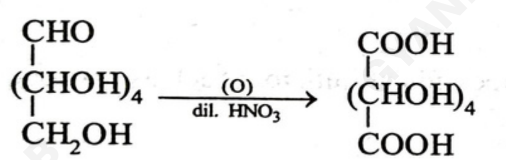
91. (A)

$$2A + B \rightarrow 3C$$

$$\text{Rate} = -\frac{1}{2}\frac{d[A]}{dt} = -\frac{d[B]}{dt} = \frac{1}{3}\frac{d[C]}{dt}$$

$$\therefore \frac{d[B]}{dt} = \frac{1}{3} \times 1.3 \times 10^{-4}\ \text{mol L}^{-1}\ \text{s}^{-1}$$
$$= 4.33 \times 10^{-5}\ \text{mol L}^{-1}\ \text{s}^{-1}$$

92. (A)



Glucose

Saccharic acid

93. (A)

$V_1 = 15\ dm^3$, $V_2 = 20\ dm^3$, $W = -6\ dm^3$ bar

$$W = -P_{ext}\,\Delta V$$

$$\therefore P_{ext} = -\frac{W}{\Delta V} = -\frac{W}{(V_2 - V_1)}$$

$$= \frac{-(-6\,dm^3\,bar)}{(20-15)\,dm^3} = \frac{6}{5}\,bar$$

$$= 1.2\ bar = 1.2 \times 10^5\ Pa$$

94. (D)

In heterolytic cleavage of covalent bond both shared electrons go to one of the two bonded atoms.

95. (B)
n-pentane is a straight chain alkane, exhibits strong London forces as compared to other branched chain alkanes.

96. (A)
$I = 5A,\ t = 20\ \text{min} = 20 \times 60 = 1200\ s$

$Q = I \times t = 5\ A \times 1200\ s$

$= 6000\ C$

Moles of electrons actually passed $= \dfrac{Q\ (C)}{965000\,(C/\text{mol}\ e^-)}$

$= \dfrac{6000}{96500}$

$= 6.22 \times 10^{-2}\ \text{mol}\ e^-$

97. (B)
Carbon monoxide molecule $\Rightarrow$:C≡O:

Formal charge of oxygen atom $= VE - NE - \dfrac{1}{2}(BE)$

$= 6 - 2 - \dfrac{1}{2}(6) \quad = +1$

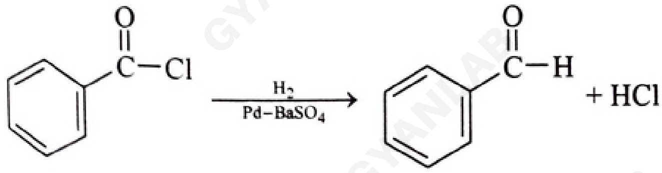
98. (A)
Straight chain isomer, i.e., hexane has highest boiling point as compared to branched chain isomers of C_6H_{14}.

99. (C)
$\nu = 50\ \text{Hz} = 50\ s^{-1},\ c = 3 \times 10^8\ m\ s^{-1}$

$\lambda = \dfrac{c}{\nu} = \dfrac{3\times10^8\ m\,s^{-1}}{50\,s^{-1}} = 6 \times 10^6\ m$

100. (D)

Resenmund Reduction



Benzoyl chloride Benzaldehyde

Section II

MATHEMATICS

101.(A)

We have $3m^2 + 10\,mn + 3n^2 = 4h^2$ ...(1)

Given lines are $mx^2 + 2hx + ny^2 = 0$

Let m_1 and m_2 be the slopes of the lines

$\therefore\ m_1 + m_2 = \frac{-2h}{n}$ and $m_1 m_2 = \frac{m}{n}$

Now $(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4\,m_1\,m_2$

$$= \left(\frac{-2h}{n}\right)^2 - \frac{4m}{n} = \frac{4h^2 - 4mn}{n^2} = \frac{3m^2 + 6mn + 3n^2}{n^2} \quad \text{...[From (1)]}$$

$$= \frac{3(m+n)^2}{n^2}$$

$\therefore\ m_1 - m_2 = \frac{\sqrt{3}\,(m+n)}{n}$

$\therefore$ Let θ be the required angle.

Now $\tan\theta = \frac{|m_1 - m_2|}{1 + m_1 m_2}$

$$= \frac{\left(\frac{\sqrt{3}\,(m+n)}{n}\right)}{\left(1+\frac{m}{n}\right)} = \frac{\sqrt{3}\,(m+n)}{n} \times \frac{n}{m+n} = \sqrt{3} \Rightarrow \theta = 60^\circ = \frac{\pi^c}{3}$$

102.(C)

We will check all 3 equations.

I : $\frac{dy}{dx} = \frac{y+x}{x} \Rightarrow$ All terms have same degree equal to 1.

II : $\frac{dy}{dx} = \frac{x^2+y}{y^3} \Rightarrow$ Here all terms have different degrees.

III : $\frac{dy}{dx} = \frac{2xy}{y^2 - x^2} \Rightarrow$ All terms have same degree equal to 2.

103.(B)

Since circles touch Y axis at origin, the centres of the circles lie on X axis.

Let centre be (h, 0) and radius = h.

$\therefore\ (x-h)^2 + (y-0)^2 = h^2 \Rightarrow x^2 - 2hx + y^2 = 0$ ...(1)

Differentiating w.r.t. x, we get

$2x - 2h + 2y\frac{dy}{dx} = 0 \quad \Rightarrow \quad x + y\frac{dy}{dx} = h$

Substituting value of h in eq. (1), we get

$$x^2 - 2\left(x + y\frac{dy}{dx}\right)x + y^2 = 0$$

$\therefore\ x^2 - 2x^2 - 2xy\frac{dy}{dx} + y^2 = 0 \quad \Rightarrow x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

104.(C)

We have $n = 5$, $\bar{x} = 4$ and $\sigma^2 = 5.2$

Let the 5 observations be 1, 2, 6, a, b

$\therefore 4 = \dfrac{1+2+6+a+b}{5} \Rightarrow a + b = 11$ ...(1)

Variance $= \dfrac{\Sigma(x_i - \bar{x})^2}{n}$

$\therefore 5.2 = \dfrac{(1-4)^2 + (2-4)^2 + (6-4)^2 + (a-4)^2 + (b-4)^2}{5}$

$\therefore 26 = 9 + 4 + 4 + (a-4)^2 + (b-4)^2 \Rightarrow (a-4)^2 + (b-4)^2 = 9$ ...(2)

From (1), we get $b = 11 - a$ and substituting value of b in (2), we write

$(a-4)^2 + (7-a)^2 = 9 \Rightarrow a = 4, 7 \Rightarrow$ (a, b) are 4 and 7.

105.(A)

$\bar{a} + \lambda\bar{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + 2\hat{k})$

$= (1+3\lambda)\hat{i} + (2-\lambda)\hat{j} + (-3+2\lambda)\hat{k}$

Since $\bar{a} + \lambda\bar{b}$ is $\perp$er to $\bar{c}$, we write

$(1)(1 + 3\lambda) + (3)(2 - \lambda) + (1)(-3 + 2\lambda) = 0$

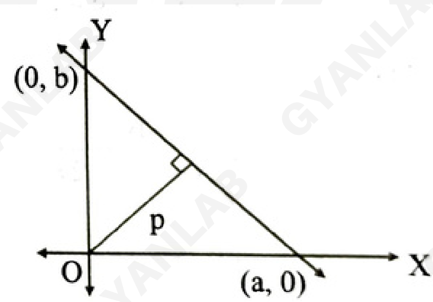
$\therefore 1 + 3\lambda + 6 - 3\lambda - 3 + 2\lambda = 0 \Rightarrow \lambda = -2$

106.(D)

Refer figure

Equation of given line is

$\dfrac{x}{a} + \dfrac{y}{b} = 1$ i.e. $bx + ay = ab$ ...(1)



Distance of line (1) from origin is

$\dfrac{|-ab|}{\sqrt{a^2+b^2}} = p \Rightarrow a^2 + b^2 = \dfrac{a^2b^2}{p^2}$

$\therefore \dfrac{a^2+b^2}{a^2b^2} = \dfrac{1}{p^2} \Rightarrow \dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{p^2}$

107.(C)

We have $y = x^3 - 3x^2 - 9x + 5$

$\therefore \dfrac{dy}{dx} = 3x^2 - 6x - 9$ and since tangent is parallel to X axis, we write

$3x^2 - 6x - 9 = 0 \Rightarrow x^2 - 2x - 3 = 0$ i.e. $(x-3)(x+1) = 0 \Rightarrow x = -1, 3$

108.(B)

Let $I = \int \dfrac{dx}{x^{\frac{1}{2}} + x^{\frac{1}{3}}}$

Put $x^{\frac{1}{6}} = t \Rightarrow x = t^6 \Rightarrow dx = 6t^5\,dt$. Also $x^{\frac{1}{2}} = t^3$ and $x^{\frac{1}{3}} = t^2$

$$\therefore\ I = 6\int \frac{t^5\,dt}{t^2(1+t)}$$

$$= 6\int \frac{t^3}{(1+t)}dt = 6\int \frac{(t^3+1)-1}{(1+t)}dt = 6\int \frac{(t+1)(t^2-t+1)}{(1+t)}dt - 6\int \frac{dt}{1+t}$$

$$= 6\int (t^2-t+1)dt - 6\log|1+t| + c = \frac{6t^3}{3} - \frac{6t^2}{2} + 6t - 6[\log|1+t|] + c$$

$$= 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log\left|1+\sqrt[6]{x}\right| + c$$

109.(C)

$$A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore\ \text{adj } A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

110.(D)

Let $I = \int [\sin|\log x| + \cos|\log x|]dx$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt \Rightarrow dx = e^t\,dt$

$$\therefore\ I = \int (\sin + \cos t)e^t\,dt$$

$$= \int e^t(\sin t + \cos t)dt = e^t \sin t + c = x\sin|\log x| + c$$

111.(D)

Let $I = \int_5^{10} \frac{dx}{(x-1)(x-2)}$

$$= \int_5^{10}\left[\frac{1}{x-1} - \frac{1}{x-2}\right](-1)\,dx = -\int_5^{10}\left[\frac{1}{x-1} - \frac{1}{x-2}\right]dx$$

$$= -[\log|x-1|]_5^{10} + [\log|x-2|]_5^{10} = -[\log|9| - \log|4|] + [\log|8| - \log 3|]$$

$$= \left[\log\left|\frac{8}{3}\right|\right] - \left[\log\left|\frac{9}{4}\right|\right] = \log\left|\frac{8}{3}\times\frac{4}{9}\right| = \log\left|\frac{32}{27}\right|$$

112.(C)

We have line $\frac{x+2}{2} = \frac{2y-5}{3}, z = -1$

i.e. $\frac{x-(-2)}{2} = \frac{2\left(y-\frac{5}{2}\right)}{3}, z = -1$ i.e. $\frac{x-(-2)}{2} = \frac{\left(y-\frac{5}{2}\right)}{\left(\frac{3}{2}\right)}, z = -1$

Here direction ratios are $2, \frac{3}{2}, 0$

Also $\sqrt{(2)^2+\left(\frac{3}{2}\right)^2+0} = \pm\frac{5}{2}$

Here required direction cosines are

$$\frac{2}{\left(\pm\frac{5}{2}\right)}, \frac{\left(\frac{3}{2}\right)}{\left(\pm\frac{5}{2}\right)}, \frac{0}{\left(\pm\frac{5}{2}\right)} \text{ i.e. } \pm\frac{4}{5}, \pm\frac{3}{5}, 0$$

113.(D)

Required probability $= \dfrac{{}^3C_1 \times {}^4C_1 \times {}^2C_1}{{}^9C_3}$

$$= \frac{3\times4\times2}{\left(\frac{9!}{3!6!}\right)} = \frac{24\times6}{9\times8\times7} = \frac{2}{7}$$

114.(C)

$|\bar{a}\times\bar{b}| = 25$

$\therefore |\bar{a}|\cdot|\bar{b}|\cdot\sin\theta = (5)(13)\sin\theta = 25$

$\therefore \sin\theta = \frac{5}{13} \Rightarrow \cos\theta = \frac{-12}{13}$ $\quad \ldots\left[\because \frac{\pi}{2}<\theta\le\pi\right]$

$\therefore \bar{a}\cdot\bar{b} = |\bar{a}|\cdot|\bar{b}|\cdot\cos\theta$

$= (5)(13)\left(\frac{-12}{13}\right) = -60$

115.(C)

Normal to the plane is perpendicular to the given vectors.

Hence equation of normal is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 2 & -1 \end{vmatrix} = \hat{i}(-1)-\hat{j}(-4)+\hat{k}(5) = -\hat{i}+4\hat{j}+5\hat{k}$$

Hence equation of required plane is

$(-1)(x-2)+(4)(y-0)+(5)(z-5)=0$

$\therefore -x+2+4y+5z-25=0 \quad \Rightarrow \quad x-4y-5z+23=0$

116.(D)

We know that $AA^{-1} = I$

$$\therefore \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 0 & c+1 \\ 0 & 1 & 2+2c \\ 4-4a & 3a-3 & 2+ac \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus $c+1=0 \Rightarrow c=-1$ and $4-4a=0 \Rightarrow a=1$

117.(A)

We have $f(x) = \dfrac{1-x+x^2}{1+x+x^2}$

$$\therefore \; f'(x) = \frac{(1+x+x^2)(2x-1)-(1-x+x^2)(2x+1)}{(1+x+x^2)^2}$$

$$= \frac{(2x+2x^2+2x^3-x-1-x^2)-(2x-2x^2+2x^3+1-x+x^2)}{(1+x+x^2)^2}$$

$$= \frac{(x+x^2+2x^3-1)-(x-x^2+2x^3+1)}{(1+x+x^2)^2}$$

$$= \frac{2x^2-2}{(1+x+x^2)^2}$$ and when $f'(x) = 0$, we get

$2(x^2-1) = 0 \quad \Rightarrow \quad x = \pm 1$

When $x = 1$, $f(x) = \dfrac{1}{3}$ and when $x = -1$, $f(x) = 3$

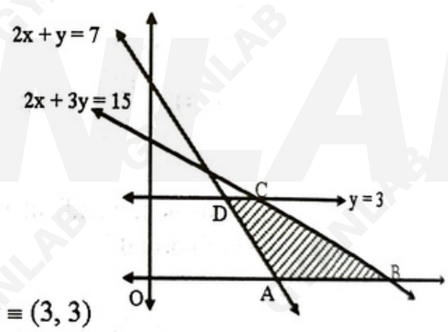
Hence minimum value of f(x) is $\dfrac{1}{3}$.

118.(B)

We have lines $2x + y = 7$, $2x + 3y = 15$, $y = 3$

Refer figure

The required region is shaded.



We have $A \equiv \left(\dfrac{7}{2}, 0\right)$, $B \equiv \left(\dfrac{15}{2}, 0\right)$

Point of intersection of $2x + y = 7$ and $y = 3$ is $D \equiv (2, 3)$

Point of intersection of $2x + 3y = 15$ and $y = 3$ is $C \equiv (3, 3)$

We have objective function $z = 4x + 5y$

$\therefore \; z_{(A)} = 4\left(\dfrac{7}{2}\right) + 5(0) = 14 + 0 = 14$

$z_{(B)} = 4\left(\dfrac{15}{2}\right) + 5(0) = 30 + 0 = 30$

$z_{(C)} = 4(3) + 5(3) = 12 + 15 = 27$

$z_{(D)} = 4(2) + 5(3) = 8 + 15 = 23$

Hence minimum value occurs at point A which lies on X axis.

119.(A)

We have $O \equiv (0, 0, 0)$ and $P \equiv (1, 2, 3)$

Hence direction ratios of $\overline{OP}$ are

$$\frac{1-0}{\sqrt{1^2+2^2+3^2}}, \frac{2-0}{\sqrt{1^2+2^2+3^2}}, \frac{3-0}{\sqrt{1^2+2^2+3^2}} = \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$$

120.(A)

Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}\,dx$ ...(1)

$= \int_{-\pi/2}^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-\frac{\pi}{2}-x\right)}{1+e^{\left(\frac{\pi}{2}-\frac{\pi}{2}-x\right)}}\,dx = \int_{-\pi/2}^{\pi/2} \frac{\cos(-x)}{1+e^{-x}}\,dx = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+\left(\frac{1}{e^x}\right)}\,dx$

$= \int_{-\pi/2}^{\pi/2} \frac{e^x \cos x}{1+e^x}\,dx$ ...(2)

Eq. (1) + (2) gives

$2I = \int_{-\pi/2}^{\pi/2} \frac{\cos x\left(1+e^x\right)}{\left(1+e^x\right)}\,dx = \int_{-\pi/2}^{\pi/2} \cos x\,dx = 2\int_0^{\pi/2} \cos x\,dx$

$\therefore I = \int_0^{\pi/2} \cos x\,dx = [\sin x]_0^{\pi/2} = 1$

121.(B)

Let a, b, c be the direction cosines of the required plane.

It contains the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and passes through the point (0, 7, –7)

$\therefore\ a(x+1) + b(y-3) + c(z+2) = 0$ ...(1)

$\therefore\ a(0+1) + b(7-3) + c(-7+2) = 0 \Rightarrow a + 4b - 5c = 0$ ...(2)

Also $-3a + 2b + c = 0$ ...(3)

From (2) and (3), we write

$$\frac{a}{\begin{vmatrix} 4 & -5 \\ 2 & 1 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 1 & -5 \\ -3 & 1 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix}}$$

$\therefore\ \frac{a}{14} = \frac{b}{14} = \frac{c}{14} \Rightarrow a = b = c = 1$

Hence eq. (1) becomes

$x + 1 + y - 3 + z + 2 = 0 \Rightarrow x + y + z = 0$

122.(A)

We have lines $x^2 - 4xy + y^2 = 0$ and slopes of the lines are $\tan\alpha$ and $\tan\beta$.

$\therefore\ \tan\alpha + \tan\beta = 4$ and $\tan\alpha \cdot \tan\beta = 1$

$\therefore\ \tan\alpha = \frac{1}{\tan\beta} = \cot\beta$

$\therefore\ \cot\beta + \tan\beta = 4$ and squaring, we get $\cot^2\beta + \tan^2\beta + 2 = 16 \Rightarrow \tan^2\beta + \cot^2\beta = 14$

123.(B)

We have probability of cases getting solved = 25% = $\frac{1}{4}$

$\therefore$ $p = \frac{1}{4} \Rightarrow q = \frac{3}{4}$ and we have n = 6, x = 5, 6

Hence required probability

$$= \left[{}^6C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^1\right] + \left[{}^6C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^0\right]$$

$$= \frac{(6)(3)}{(4)^6} + \frac{1}{(4)^6} = \frac{19}{(4)^6} = \frac{19}{4096}$$

124.(C)

We have p = 0.6 and np = 6 $\Rightarrow$ n = 10

$\therefore$ Var (X) = npq = (10) (0.6) (0.4) = 2.4

125.(A)

Let a, b, c be the direction ratios of the required line.

$\therefore$ $2a - 2b + c = 0$ ...(1) and $a - 2b + 2c = 0$ ...(2)

From (1) and (2), we write

$$\frac{a}{\begin{vmatrix}-2 & 1\\ -2 & 2\end{vmatrix}} = \frac{-b}{\begin{vmatrix}2 & 1\\ 1 & 2\end{vmatrix}} = \frac{c}{\begin{vmatrix}2 & -2\\ 1 & -2\end{vmatrix}} \Rightarrow \frac{a}{-2} = \frac{b}{-3} = \frac{c}{-2}$$

$\therefore$ (a, b, c) = (2, 3, 2)

So equation of required line is

$$\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$$

126.(C)

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sqrt{1+px} - \sqrt{1-px}}{x}$$

$$= \lim_{x\to 0^{-1}} \frac{\left[\left(\sqrt{1+px}\right) - \left(\sqrt{1-px}\right)\right]\left[\left(\sqrt{1+px}\right) + \left(\sqrt{1-px}\right)\right]}{x\left[\left(\sqrt{1+px}\right) + \left(\sqrt{1-px}\right)\right]}$$

$$= \lim_{x\to 0^-} \frac{\left[(1+px) - (1-px)\right]}{x\left[\sqrt{1+px} + \sqrt{1-px}\right]} = \lim_{x\to 0^-} \frac{2p}{\sqrt{1+px} + \sqrt{1-px}}$$

$$= \frac{2p}{2} = p$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{2x+1}{x-2} = \frac{1}{-2}$$

Since f(x) is continuous at x = 0, we get $p = \frac{-1}{2}$

127.(D)

$f(x) = \log(1+x) - \frac{2x}{2+x} \Rightarrow x \neq -2$

$\therefore f'(x) = \frac{1}{(1+x)} - \left[\frac{(2+x)(2)-(2x)(1)}{(2+x)^2}\right]$

$= \frac{1}{1+x} - \left[\frac{4}{(2+x)^2}\right] = \frac{(x+2)^2 - 4(x+1)}{(x+2)^2(1+x)}$

When $f'(x) > 0$, we write

$\frac{x^2}{(x+2)^2(1+x)} > 0$

Since $x^2 > 0$ and $(x+2)^2 > 0$, we write $(1+x) > 0 \Rightarrow x > -1$

128.(B)

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 + R_2, R_3 \to R_3 - 3R_2$

$$\begin{bmatrix} 1 & 3 & 5 \\ 1 & 2 & 3 \\ 0 & -5 & -8 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & -1 & -2 \\ 0 & -5 & -8 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

$R_1 \to R_1 + 3R_2$ and $R_3 \to R_3 - 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -2 \\ 0 & 0 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ -1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix}$$

$R_2 \to R_2 + R_3$ and $R_1 \to 2R_1 + R_3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ 4 & -3 & 1 \\ 5 & -3 & 1 \end{bmatrix}$$

$R_1 \to \frac{1}{2}R_1, R_2 \to -R_2, R_3 \to \frac{1}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$$

129.(A)

We have $2\pi r = 10\pi \Rightarrow r = 5$ and centre of circle is (2, –3)

Hence equation of circle is

$(x-2)^2 + (y+3)^2 = (5)^2$ i.e. $x^2 + y^2 - 4x + 6y - 12 = 0$

130.(D)

$y = 2\sin x + 3\cos x$

$\therefore \dfrac{dy}{dx} = 2\cos x - 3\sin x$

$\therefore \dfrac{d^2y}{dx^2} = -2\sin x - 3\cos x = -(2\sin x + 3\cos x) = -y$

$\therefore y + \dfrac{d^2y}{dx^2} = 0$

We have $y + A\dfrac{d^2y}{dx^2} = B \Rightarrow A = 1, B = 0$

131.(A)

$\cos 2\theta = \sin\theta$

$\therefore 1 - 2\sin^2\theta = \sin\theta \quad \Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$

$\therefore (2\sin\theta - 1)(\sin\theta + 1) = 0 \quad \Rightarrow \sin\theta = \dfrac{1}{2}, -1$

When have $\theta \in (0, 2\pi)$

$\therefore$ Possible values of θ are $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$

132.(B)

$$y = \tan^{-1}\left[\frac{1}{1+x+x^2}\right] + \tan^{-1}\left[\frac{1}{x^2+3x+3}\right]$$

$$= \tan^{-1}\left[\frac{1}{1+x(1+x)}\right] + \tan^{-1}\left[\frac{1}{1+(x+2)(x+1)}\right]$$

$$= \tan^{-1}\left[\frac{(x+1)-(x)}{1+(x+1)(x)}\right] + \tan^{-1}\left[\frac{(x+2)-(x+1)}{1+(x+2)(x+1)}\right]$$

$= \tan^{-1}(x+1) - \tan^{-1}(x) + \tan^{-1}(x+2) - \tan^{-1}(x+1)$

$= \tan^{-1}(x+2) - \tan^{-1}(x)$

$$\therefore \frac{dy}{dx} = \frac{1}{1+(x+2)^2} - \frac{1}{1+x^2}$$

133.(A)

We have $(x + iy)(2 - i) = 3 + i$

$\therefore 2x + (2y - x)i + y = 3 + i$

$\therefore 2x + y = 3 \quad \ldots(1)$ and $2y - x = 1 \quad \ldots(2)$

Solving eq. (1) and (2), we get

$x = 1, y = 1 \Rightarrow z = 1 + i = \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$\therefore\ z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\therefore\ z^{38} = \left(\sqrt{2}\right)^{38}\left(\cos\frac{38\pi}{4} + i\sin\frac{38\pi}{4}\right)$

$= (2)^{19}\left[\cos\left(9\pi + \frac{\pi}{2}\right) + i\sin\left(9\pi + \frac{\pi}{2}\right)\right]$

$= (2)^{19}\left(-\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right) = 2^{19}(0 - i)$

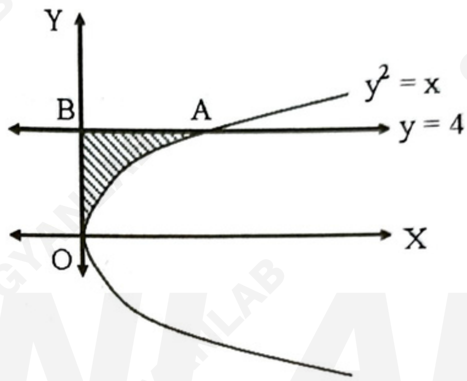
$= -\left(2^{19}\right)i$

134.(B)

Refer figure

Required area is shaded.

Point of intersection of $y^2 = x$ and $y = 4$ is $A \equiv (16, 4)$.



$\therefore\ A = \int_0^4 y^2 dy$

$= \left[\frac{y^3}{3}\right]_0^4 = \frac{64}{3}$ sq. units

135.(A)

We have $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\alpha$

$\therefore\ \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right) = \tan^{-1}\left(\frac{\alpha}{\sqrt{1-\alpha^2}}\right)$

$\therefore\ \tan^{-1}\left[\frac{\left(\frac{3}{4}\right) + \left(\frac{5}{12}\right)}{1 - \left(\frac{3}{4}\right)\left(\frac{5}{12}\right)}\right] = \tan^{-1}\left(\frac{\alpha}{\sqrt{1-\alpha^2}}\right)$

$\therefore\ \tan^{-1}\left[\frac{\left(\frac{14}{12}\right)}{\left(\frac{11}{16}\right)}\right] = \tan^{-1}\left(\frac{\alpha}{\sqrt{1-\alpha^2}}\right)$

$\therefore\ \frac{56}{33} = \frac{\alpha}{\sqrt{1-\alpha^2}} \Rightarrow (56)^2(1 - \alpha^2) = (33)^2\alpha^2$

$\therefore\ \alpha^2 = \frac{(56)^2}{(56)^2 + (33)^2} = \frac{(56)^2}{(65)^2} \Rightarrow \alpha = \frac{56}{65}$

136.(C)

$$\lim_{x\to 5}\frac{x^k-5^k}{x-5}=500$$

$\therefore\ (k)(5)^{k-1} = 500$

$= 4(125) \quad = 4(5)^3 \quad = 4(5)^{4-1}$

$\therefore\ k = 4$

137.(A)

$(p \to q) \wedge (q \to \sim p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$

$= (\sim p) \vee (q \wedge \sim q)$

$\equiv (\sim p) \vee F \equiv \sim p$

138.(B)

Each of the five questions can be solved in two ways.

$\therefore$ Maximum number of wrong answers.

$= (2 \times 2 \times 2 \times 2 \times 2) - 1 = 31$

139.(D)

We have $\left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right)dx - \left[\left(\frac{x}{y}\right)\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right]dy = 0$

$$\therefore\ \frac{dy}{dx}=\frac{\left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right)}{\left(\frac{x}{y}\right)\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)}$$

Put $\frac{y}{x}=v \Rightarrow y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\therefore\ v+x\frac{dv}{dx} = \frac{v\cos v}{\frac{1}{v}\sin v+\cos v}=\frac{v^2\cos v}{\sin v+v\cos v} \Rightarrow x\frac{dv}{dx}=\frac{-v\sin v}{\sin v+v\cos v}$$

$$\therefore\ \int\frac{\sin v+v\cos v}{v\sin v}dv=\int\frac{-dx}{x} \Rightarrow \int\frac{1}{v}dv+\int\cot v\,dv = -\int\frac{dx}{x}$$

$\therefore\ \log|v| + \log|\sin x| = -\log|x| + \log k \quad \Rightarrow \log|(v)(\sin v)(x)| = \log k$

$\therefore\ y\sin\left(\frac{y}{x}\right) = k$

140.(A)

Half life period = 5 years.

Initial quantity of substance = 64 grams

$\therefore$ Quantity left after 5 years $= \frac{64}{2} = 32$ grams

$\therefore$ Quantity left after 10 years $= \frac{32}{2} = 16$ grams

$\therefore$ Quantity left after 15 years $= \frac{16}{2} = 8$ grams

141.(A)

Expected value = $\Sigma p_i x_i$

$= (2)\left(\frac{20}{100}\right)+(2.5)\left(\frac{5}{100}\right)+(3)\left(\frac{10}{100}\right)+(1.5)\left(\frac{50}{100}\right)+(1)\left(\frac{15}{100}\right)$

$= 0.4 + 0.125 + 0.3 + 0.75 + 0.15 = 1.725$

142.(B)

We have $\left(\frac{d^2y}{dx^2}\right)^5 + \frac{4\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{d^3y}{dx^3}\right)} + \left(\frac{d^3y}{dx^3}\right) = x^2 - 1$

$\therefore \left(\frac{d^3y}{dx^3}\right)\left(\frac{d^2y}{dx^2}\right)^5 + 4\left(\frac{d^2y}{dx^2}\right) + \left(\frac{d^3y}{dx^3}\right)^2 = (x^2-1)\left(\frac{d^3y}{dx^3}\right)$

$\therefore$ order = 3, degree = 2

143.(C)

We have $\tan\theta = k\tan\phi$ and $\theta + \phi = \alpha$

$\therefore \frac{\tan\theta}{\tan\phi} = \frac{k}{1}$

By Componendo Dividendo, we get

$\frac{\tan\theta + \tan\phi}{\tan\theta - \tan\phi} = \frac{k+1}{k-1}$

$\therefore \frac{\frac{\sin\theta}{\cos\theta} + \frac{\sin\phi}{\cos\phi}}{\frac{\sin\theta}{\cos\theta} - \frac{\sin\phi}{\cos\phi}} = \frac{k+1}{k-1}$

$\therefore \frac{\sin\cos\phi + \cos\theta\sin\phi}{\sin\theta\cos\phi - \cos\theta\sin\phi} = \frac{k+1}{k-1}$

$\therefore \frac{\sin(\theta+\phi)}{\sin(\theta-\phi)} = \frac{k+1}{k-1} \quad \Rightarrow \quad \frac{\sin\alpha}{\sin(\theta-\phi)} = \frac{k+1}{k-1}$

$\therefore \sin(\theta - \phi) = \frac{k-1}{k+1}(\sin\alpha)$

144.(D)

Let $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

$\therefore \sin A = \frac{a}{k}, \sin B = \frac{b}{k}, \sin C = \frac{c}{k}$

With usual notations, from the given data, we write

$$a + b + c = 6\left[\frac{\left(\frac{a}{k}+\frac{b}{k}+\frac{c}{k}\right)}{3}\right]$$

$\therefore (a+b+c) = \frac{2(a+b+c)}{k} \quad \Rightarrow \quad k = 2$

$\therefore \sin A = \frac{a}{k} = \frac{1}{2} \quad \Rightarrow A = \frac{\pi^c}{6}$

145.(A)

$p \to (\sim p \vee q) \equiv F$

We know that $T \to F \equiv F$

$\therefore$ $p \equiv T$ and $(\sim p \vee q) \equiv F$.

We know that $F \vee F \equiv F$

$\therefore$ $\sim p \equiv F$ and $q \equiv F$

Thus p, q are T, F respectively.

146.(C)

$|\bar{a} \times \bar{b}|^2 + (\bar{a} \cdot \bar{b})^2 = 144$ and $|\bar{a}| = 4$

$\therefore$ $\left(|a|^2 \cdot |b|^2 \cdot \sin^2\theta\right) + \left(|\bar{a}|^2 |\bar{b}|^2 \cos^2\theta\right) = 144$

$\therefore$ $|\bar{a}|^2 |\bar{b}|^2 \left(\cos^2\theta + \sin^2\theta\right) = 144$

$\therefore$ $(4)^2 |\bar{b}|^2 = 144 \Rightarrow |\bar{b}|^2 = 9 \Rightarrow |\bar{b}| = 3$

147.(C)

We have A = {10, 11, 12, 14, 26} and

$f : A \to N$, where f(a) = Highest prime factor of a, when $a \in A$.

Now $10 = 2 \times 5 \Rightarrow$ Highest prime factor is 5

$11 = 1 \times 11 \Rightarrow$ Highest prime factor is 11

$12 = 2 \times 2 \times 3 \Rightarrow$ Highest prime factor is 3

$14 = 2 \times 7 \Rightarrow$ Highest prime factor is 7

$26 = 2 \times 13 \Rightarrow$ Highest prime factor is 13

$\therefore$ f = {3, 5, 7, 11, 13}

148.(D)

Let $I = \int \frac{5 \tan x}{\tan x - 2} dx$

$I = \int \frac{5 \sin x}{\sin x - 2 \cos x} dx$

Here $\frac{d}{dx}(\sin x - 2\cos x) = \cos x + 2 \sin x$

$$\therefore I = \int \frac{(2\sin x + 2\sin x + \sin x) + (2\cos x - 2\cos x)}{\sin x - 2\cos x} dx$$

$$= \int \frac{(2\sin x + \cos x) + (2\sin x + \cos x) + (\sin x - 2\cos x) dx}{\sin x - 2\cos x}$$

$$= \int \frac{2(2\sin x + \cos x) + (\sin x - 2\cos x)}{\sin x - 2\cos x} dx$$

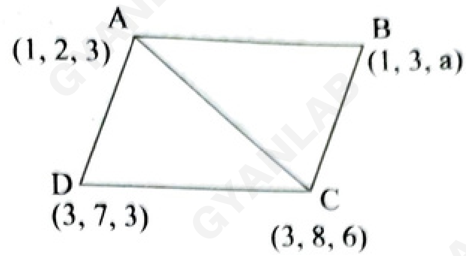
$$= \int dx + 2\int \frac{2\sin x + \cos x}{\sin x - 2\cos x} dx$$

$$= x + 2\log|\sin x - 2\cos x| + c$$

From given data, a = 2

149.(B)

Refer figure



$$A(\Delta ABC) = \frac{A(\square ABCD)}{2}$$

$$= \frac{\sqrt{265}}{2}$$

Also $A(\Delta\, ABC) \quad = \frac{1}{2}|\overline{BA} \times \overline{BC}|$

Here $\overline{BA} \quad = -\hat{j} + (3-a)\hat{k}$ · and $\overline{BC} = 2\hat{i} + 5\hat{j} + (6-a)\hat{k}$

Now $\overline{BA} \times \overline{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & (3-a) \\ 2 & 5 & (6-a) \end{vmatrix}$

$$= \hat{i}(-6+a-15+5a) - \hat{j}(-6+2a) + \hat{k}(2)$$

$$= (6a-21)\hat{i} - (2a-6)\hat{j} + 2\hat{k}$$

Magnitude of $\overline{BA} \times \overline{BC} = \sqrt{(6a-21)^2 + (2a-6)^2 + 4}$

$\therefore \ \frac{1}{2}\sqrt{(6a-21)^2 + (2a-6)^2 + 4} \quad = \frac{\sqrt{265}}{2}$

$\therefore \ (6a-21)^2 + (2a-6)^2 + 4 = 265$

$\therefore \ 40\,a^2 - 276\,a + 216 = 0$

$\therefore \ 10a^2 - 69a + 54 = 0$

$\therefore \ a \ = \frac{69 \pm \sqrt{(69)^2 - (4)(54)(10)}}{20} = \frac{69 \pm 51}{20} = \frac{120}{20}$ or $\frac{18}{20}$

$\therefore \ a = 6$ from options given.

150.(C)

$$y \ = \tan^{-1}\left\{\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right\}$$

Put $a = r\cos\alpha$, $b = r\sin\alpha$

$$\therefore \ y \ = \tan^{-1}\left\{\frac{r(\cos x\cos\alpha - \sin x\sin\alpha)}{r(\sin\alpha\cos x + \cos\alpha\sin x)}\right\}$$

$$= \tan^{-1}\left[\frac{\cos(x+\alpha)}{\sin(x+\alpha)}\right] = \tan^{-1}[\cot(x+\alpha)]$$

$$= \tan^{-1}\left\{\tan\left[\frac{\pi}{2} - (x+\alpha)\right]\right\} = \frac{\pi}{2} - (x+\alpha)$$

$$\therefore \ \frac{dy}{dx} \ = 0 - (1+0) = -1$$